8. $y = \sqrt{1+e^x}$, $y' = \dfrac{e^x}{2\sqrt{1+e^x}}$

$$S = \int_0^1 2\pi\sqrt{1+e^x}\sqrt{1+\frac{e^{2x}}{4(1+e^x)}}\,dx$$

$$= \pi\int_0^1 \cancel{2\sqrt{1+e^x}}\sqrt{\frac{4+4e^x+e^{2x}}{\cancel{4(1+e^x)}}}\,dx$$

$$= \pi\int_0^1 \sqrt{(2+e^x)^2}\,dx = \int_0^1 \pi(2+e^x)\,dx$$

$$= \pi(2x+e^x\Big|_0^1 = \pi\left[(2+e)-(0+1)\right] = \boxed{\pi(1+e)}$$

10. $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$, $y' = \dfrac{x^2}{2} - \dfrac{1}{2x^2}$

$$S = \int_{1/2}^1 2\pi\left(\frac{x^3}{6}+\frac{1}{2x}\right)\sqrt{1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2}\,dx$$

$$= \int_{1/2}^1 2\pi\left(\frac{x^3}{6}+\frac{1}{2x}\right)\sqrt{\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2}\,dx$$

$$= 2\pi\int_{1/2}^1 \left(\frac{x^3}{6}+\frac{1}{2x}\right)\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)dx$$

$$= \pi\int_{1/2}^1\left(\frac{x^5}{6}+\frac{x}{2}+\frac{x}{6}+\frac{1}{2x^3}\right)dx = \pi\int_{1/2}^1\left(\frac{x^5}{6}+\frac{2}{3}x+\frac{1}{2x^3}\right)dx$$

$$= \pi\left[\frac{x^6}{36}+\frac{x^2}{3}-\frac{1}{4x^2}\right|_{1/2}^1 = \frac{263}{256}\pi$$

26. $S = \int_0^\infty 2\pi e^{-x}\sqrt{1+e^{-2x}}\,dx$

Let $u = e^{-x}$
$du = -e^{-x}dx$
$-du = e^{-x}dx$

x	u
0	1
∞	0

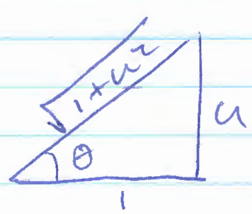
$$= 2\pi\int_1^0 \sqrt{1+u^2}\,(-du)$$

$$= 2\pi\int_0^1 \sqrt{1+u^2}\,du$$

$$= 2\pi\int_0^{\pi/4}\sec^3\theta\,d\theta$$

Let $u = \tan\theta$
$du = \sec^2\theta\,d\theta$
$\sqrt{1+u^2} = \sec\theta$

u	θ
0	0
1	$\pi/4$



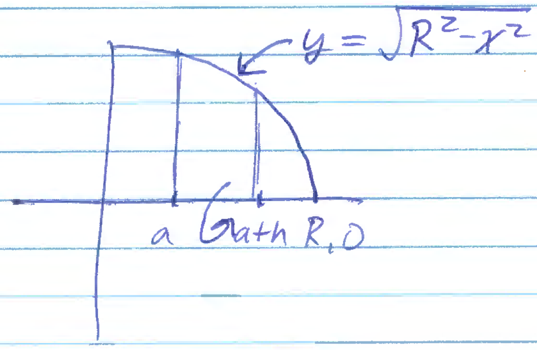
26. (continued)

$$S = 2\pi \int_0^{\pi/4} \sec^3\theta \, d\theta$$

$$= \cancel{2}\pi \left[\cancel{\tfrac{1}{2}} \left(\sec\theta \tan\theta + \ln|\sec\theta + \tan\theta| \right) \right]_0^{\pi/4}$$

$$= \boxed{\pi \left[\sqrt{2} + \ln(\sqrt{2}+1) \right]}$$

34. $y = \sqrt{R^2 - x^2}$



a) $y' = \dfrac{-x}{\sqrt{R^2-x^2}}$

$$S = \int_a^{a+h} 2\pi \sqrt{R^2-x^2} \sqrt{1 + \frac{x^2}{R^2-x^2}} \, dx$$

$$= 2\pi \int_a^{a+h} \sqrt{R^2-x^2} \sqrt{\frac{(R^2-x^2)+x^2}{R^2-x^2}} \, dx$$

$$= 2\pi \int_a^{a+h} R \, dx = 2\pi R x \Big|_a^{a+h} = \boxed{2\pi R h}$$

b) Part (b) can be done without calculus. Just imagine cutting the cylinder lengthwise and laying it out into a rectangle with sides $2\pi R$ and h, respectively. Then the surface area of the cylinder equals the area of the rectangle, i.e., $\boxed{S = 2\pi R h}$